EARTHQUAKE RESISTANT DESIGN OF BUILDINGS USING VFPI

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ABSTRACT

Recent earthquakes affecting urban areas have clearly demonstrated the vulnerability of urban building stock to ground motions. A large number of engineered buildings designed and constructed using modern techniques have been damaged. Several modern aseismic design techniques using base isolation have been proposed in published literature, some of which have also been implemented. Most isolation techniques have been found ineffective for near-source ground motions. A recently proposed sliding isolation system, known as variable frequency pendulum isolator (VFPI) has unique characteristics that help overcome the limitations of traditional isolation system for near source ground motions. The VFPI incorporates both isolation as well as restoring force mechanisms and has additional advantages due to response-dependent variable frequency of oscillation. Isolator parameters of VFPI can be chosen to obtain the desired rate of time-period variation as well as the initial time period.

In this paper, behaviour of structures isolated using VFPI subjected to near source ground motions has been investigated. It is found that while traditional isolation systems are of limited effectiveness in reducing the response of structures, structures isolated with VFPI show significant reduction in response.

1. INTRODUCTION

Use of base isolation systems has emerged as a very effective technique for aseismic design of structures. In base isolation technique, a flexible layer (or isolator) is placed between the structure and its foundation such that relative deformations are permitted at this level. Due to flexibility of the isolator layer, the time period of motion of the isolator is relatively long; resulting in shift of fundamental period of the structure away from the predominant periods of ground excitation. Extensive review of base isolation systems and their applicability is available in published literature (Buckle and Mayes, 1990; Kelly, 1986, 1993; Naeim and Kelly, 1999).

Practical isolation devices typically also include energy dissipating mechanism in order to reduce deformations at the isolator level. Friction type base isolators have been found to be very effective in reducing structural response (Mostaghel et al., 1983). The performance of friction isolators is relatively insensitive to variations in the frequency content and amplitude of the input excitation, making performance of sliding isolators very robust. Pure-Friction (PF) system, consisting of horizontal sliding surface, may experience large sliding and residual displacements, which are often difficult to incorporate in structural design. An effective mechanism to provide restoring force by gravity has been utilised in Friction Pendulum System (FPS) (Zayas et al., 1987). In this system, the sliding surface takes a concave spherical shape so that the sliding and re-centring mechanisms are integrated in one unit.

The authors have recently developed a new isolation device called the Variable Frequency Pendulum Isolator (VFPI) that incorporates the advantages of both the FPS and PF isolators (Pranesh and Sinha, 1998, 2000). The most important properties of this system are: (1) its time period of oscillation depends on sliding displacement, and (2) its restoring force has a bounded value and exhibits softening behaviour. Recent investigations systems have shown VFPI to be very effective for a variety of excitation and structural characteristics.

The study of structural response subjected to near-field ground motions has great significance since near-field ground motions are characterised by pulse type excitations having narrow range of frequencies. Behaviour of most base isolated structures subjected to near-field ground motions is not found satisfactory. In the present paper the performance of VFPI for aseismic design of multi-degree-of-freedom (MDOF) structures subjected to near-field ground motions has been investigated. The effectiveness of VFPI in comparison with the other frictional base isolation systems has been examined.

2. VFPI DESCRIPTION

Consider the motion of a rigid block of mass m sliding on a smooth curved surface of defined geometry, $y = f(x)$ as shown in Fig. 1. At any instant the horizontal restoring force due to weight of the structure is given by

$$f_R = mg\frac{dy}{dx} \tag{1}$$

Assuming that the restoring force is mathematically represented by an equivalent non-linear mass-less horizontal spring, the spring force can be expressed as the product of the equivalent spring stiffness and the deformation. Further the stiffness can be expressed as product of mass and square of isolator frequency, i.e.,

$$f_R = m\omega_b^2(x)x \tag{2}$$

Here, $\omega_b(x)$ is the instantaneous isolator frequency, and depends solely on the geometry of sliding surface. The geometry of VFPI has been derived from the geometry of an ellipse so as to get sliding isolator with desired properties (Pranesh, 2000). The expression for geometry of sliding surface of VFPI is expressed as

$$y = b\left[1 - \frac{\sqrt{d^2 + 2d\,x\,\mathrm{sgn}(x)}}{d + x\,\mathrm{sgn}(x)}\right] \tag{3}$$

where sgn(x) is the signum function which assumes a value of +1 for positive sliding displacement and −1 for negative sliding displacement.

The slope at any point on this sliding surface is given as

$$\frac{dy}{dx} = \frac{bd}{(d + x\,\mathrm{sgn}(x))^2\sqrt{d^2 + 2dx\,\mathrm{sgn}(x)}}\,x \tag{4}$$

To simplify the notations, a non-dimensional parameter $r = x\,\mathrm{sgn}(x)/d$ is used. By substituting r and the initial frequency $\omega_I^2 = gb/d^2$ in (4), and combining with (1) and (2), the isolator frequency at any sliding displacement can be expressed as

$$\omega_b^2(x) = \frac{\omega_I^2}{(1+r)^2\sqrt{1+2r}} \tag{5}$$

In the above equations, parameters b and d completely define the isolator characteristics. It can be observed that the ratio b/d^2 governs the initial frequency of the isolator. Similarly, the value of $1/d$ determines the rate of variation of isolator frequency, and this factor has been defined as frequency variation factor (FVF). The variation of oscillation frequency of a typical VFPI with respect to the sliding displacement is shown in Fig. 2(a). For comparison purposes, the oscillation frequency of FPS with same initial frequency has also been shown, which is found to be almost constant. From this plot it is seen that the oscillation frequency of VFPI sharply decreases with increasing sliding displacement and asymptotically approaches zero. The force-deformation curves for example FPS and VFPI are shown in Fig. 2(b). It can be observed that the isolator force in VFPI first increases to reach its maximum value, and later slowly decreases so as to asymptotically approach the frictional force at large sliding displacement. This is an important property of VFPI, which limits the force transmitted to the structure.

3. MATHEMATICAL FORMULATION

Consider an N-storey shear structure isolated by sliding type isolator. The motion of the structure can be in either of two phases: non-sliding phase and sliding phase. In non-sliding phase, the structure behaves like a conventional fixed base structure since there is no relative motion at the isolator level. When the frictional force at the sliding surface is overcome, there is relative motion at the sliding surface, and the structure enters sliding phase. The total motion consists of a series of alternating non-sliding and sliding phases.

3.1 Non-sliding Phase

In non-sliding phase the structure behaves as a fixed-base structure, since there is no relative motion between the ground and base mass. The equations of motion in this phase are:

$$\mathbf{M_0}\ddot{\mathbf{x}}_\mathbf{0} + \mathbf{C_0}\dot{\mathbf{x}}_\mathbf{0} + \mathbf{K_0}\mathbf{x_0} = -\mathbf{M_0}\mathbf{r_0}\ddot{x}_g \tag{6}$$

and

$$x_b = \text{constant};\ \dot{x}_b = \ddot{x}_b = 0 \tag{7}$$

where, $\mathbf{M_0}$, $\mathbf{C_0}$ and $\mathbf{K_0}$ are the mass, damping and stiffness matrices of the fixed-base structure, respectively, $\mathbf{x_0} = [x_1, x_2, \cdots, x_N]^T$ is the vector of displacements of the degrees of freedom (DOFs) of the superstructure relative to the base mass (excluding the DOF of base mass), x_b is the displacement of the base mass (m_b) relative to the ground, x_g is the ground displacement, $\mathbf{r_0}$ is the influence coefficient vector and over-dot indicates derivative with respect to time. Since the base mass does not move relative to the ground, the velocity and acceleration of the base relative to the ground are zero. However the sliding displacement may be non-zero. The structure is classically damped in this phase and hence (6) can be readily solved by usual modal analysis procedures (Clough and Penzien, 1993).

3.2 Initiation of Sliding Phase

When the structure is subjected to base excitation, it will remain in non-sliding phase unless the frictional resistance at the sliding surface is overcome. Therefore the condition for the beginning of sliding phase can be written as

$$\left|\sum_{i=1}^{N} m_i\left(\ddot{x}_i + \ddot{x}_g\right) + m_b x_g + m_t\omega_b^2(x)x_b\right| \geq m_t\mu g \tag{8}$$

3.3 Sliding Phase

Once the inequality (8) is satisfied the structure enters sliding phase and the degree of freedom (DOF) corresponding to the base mass also experiences motion. The equations of motion are now given by

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\mathbf{r}\ddot{x}_g - \mathbf{r}\mu_f \tag{9}$$

where, $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are the modified mass, damping and stiffness matrices of order N+1, $\mathbf{r}$ is the modified influence coefficient vector and μ_f is the frictional force as given below.

$$M = \begin{vmatrix} M_0 & M_O r_0 \\ [M_O r_0]^T & m_t \end{vmatrix},\quad C = \begin{vmatrix} C_0 & 0 \\ 0 & 0 \end{vmatrix},$$

$$K = \begin{vmatrix} K_0 & 0 \\ 0 & k_b \end{vmatrix},\quad \mathrm{x} = \begin{vmatrix} \mathrm{x}_0 \\ \chi_b \end{vmatrix},\quad \mathrm{r} = \begin{vmatrix} 0 \\ 1 \end{vmatrix} \text{ and } \mu_f = m_t g\mu\,\mathrm{sgn}(\dot{x}_b) \tag{10}$$

Equation (9) can be solved numerically. But for large size problems the computational effort is large and the analysis does not provide proper insight into the behaviour of the structure. In view of this and the nonclassical nature of damping, complex modal analysis is used in the present investigations.

3.4 Direction of Sliding

The direction of sliding depends on the signum function that in turn depends on the forces acting on the structure at the end of the previous non-sliding phase. Once inequality (8) is satisfied, the structure starts sliding in a direction opposite to the direction of the sum of total inertia force and restoring force at the isolator level. So, we have

$$\operatorname{sgn}(\dot{x}_b) = -\frac{\sum_{i=1}^{N} m_i\left(\ddot{x}_i + \ddot{x}_b + \ddot{x}_g\right) + m_b\left(\ddot{x}_b + \ddot{x}_g\right) + m_t\omega_b^2 x_b}{\left|\sum_{i=1}^{N} m_i\left(\ddot{x}_i + \ddot{x}_b + \ddot{x}_g\right) + m_b\left(\ddot{x}_b + \ddot{x}_g\right) + m_t\omega_b^2 x_b\right|} \tag{11}$$

The signum function remains unchanged in a particular sliding phase. The end of a sliding phase is governed by the condition that the sliding velocity of the base mass is equal to zero, i.e.,

$$\dot{x}_b = 0 \tag{12}$$

Once the sliding velocity is zero, the structure may enter a non-sliding phase, reverse its direction of sliding, or have a momentary stop and then continue in the same direction. If inequality (8) is satisfied at the same instant of time when the sliding velocity is zero, it shows that there is a sudden stop at that instant.

4. ENERGY BALANCE

Base isolators reduce structural response by filtering the seismic excitations and by dissipating energy thereby reducing the energy that needs to be dissipated by the structure. Often it is very difficult to decide a proper trade-off between structural deformations and isolator displacements for determination of isolator properties. The energy quantities are convenient to consider since they involve all the response quantities and hence represent overall response of the structure. So, the energy quantities can represent the isolator performance in a more unified manner and can be used to decide the overall performance of the isolator.

The energy balance at any instant can be derived by considering the total work done by all conservative and non-conservative forces up to that instant. The differential work done by all the forces during a small deformation of the structure $\mathbf{dx_0}$ is calculated, and then integrated to get the total work done. The final expression for the energy balance is found to be (Pranesh, 2000)

$$\begin{aligned} &\frac{1}{2}\dot{\mathbf{x}}_{\mathbf{0_t}}^{\mathbf{T}}\mathbf{M_0}\dot{\mathbf{x}}_{\mathbf{0_t}} + \frac{1}{2}m_b\dot{x}_b^{t\,2} + m_t g y + \frac{1}{2}\mathbf{x_0^T K_0 x_0} + \int[\dot{\mathbf{x}}_\mathbf{0}^\mathbf{T}\mathbf{C_0}\dot{\mathbf{x}}_\mathbf{0}]\,dt + \\ &\int m_t \mu g \operatorname{sgn}(\dot{x}_b)\,dx_b = \int[\ddot{\mathbf{x}}_\mathbf{0}^\mathbf{T}\mathbf{M_0 r_0}]\,dx_g + \int m_b \ddot{x}_{b_t}\,dx_g \end{aligned} \tag{13}$$

Equation (13) is similar to the absolute energy equation derived for the conventional MDOF structure in published literature, except for the additional terms involving the potential energy due to rising of the structure along the curved surface (third term in (13)) and the non-conservative energy term due to friction (sixth term in (13) (Uang and Bertero, 1990). There is no energy dissipation due to sliding friction during the non-sliding phase. Equation (13) can be written in short as

$$E_k + E_r + E_s + E_\xi + E_\mu = E_i \qquad (14)$$

where E_k is the sum of absolute kinetic energies of all the masses, E_r and E_s are the restorable potential energy due to rising of the structure along the sliding surface of the isolator and elastic energy due to structural deformations, respectively. E_ξ and E_μ are the energy dissipated due to structural damping and sliding friction respectively. As the sum of frictional force and the restoring force is identical to the total inertia force, the term E_i on RHS is the absolute input energy.

5. RESPONSE OF EXAMPLE STRUCTURE

The effectiveness of VFPI to reduce response of an example MDOF structure subjected to near-field earthquake excitations has been presented in this section. The example structure is a five-storey shear structure. The example building is represented as a lumped mass model with equal lumped mass of 60080 kg and equal storey stiffness of 112600 kN/m for each floor. The frequencies and modal properties for the fixed-base and isolated structures are given in Table 1. Since the natural frequencies of a structure isolated by VFPI change continuously with the isolator sliding displacement, the frequencies shown in Table 1 indicate the upper bound on the frequencies when the isolator displacement is zero.

Table 1: Modal properties of fixed-base and isolated example structures.

Mode	Isolator	1	2	3	4	5
Fixed – Freq. (Hz)	-	1.96	5.72	9.02	11.59	13.22
Eff. Modal Mass (%)	-	87.95	8.72	2.42	0.75	0.16
Isolated – Freq. (Hz)	0.49	3.64	6.92	9.76	11.93	13.31
Eff. Modal Mass (%)	99.93	0.07	0.00	0.00	0.00	0.00

The example structure is analysed for ten near field ground motions. The details of the ground motions are presented in Table 2. These ground motions represent a wide range of recorded ground motions having different peak ground acceleration (PGA), frequency composition and duration. Using the formulation presented in the paper, time history analysis is carried out for the structure. The VFPI chosen in this study has an initial isolator time period of 2.0 s and FVF equal to 5.0 per m. The corresponding values of isolator parameters *b* and *d* are 0.04 m and 0.20 m, respectively. To investigate the effectiveness of VFPI, the responses are compared with those of structure isolated with FPS and PF isolators. The FPS has been chosen with radius of 1.0 m so that its time period is around 2.0 s (equal to the initial period of the example VFPI). Coefficient of friction is assumed to be equal to 0.02. The structural damping is assumed as 5% of critical for all modes.

Table 2: Details of earthquake records used in this study.

Sr. No.	Name of earthquake	Magnitude	Distance of Source (*km*)	PGA (*g*)	Duration (*sec*)
1.	Tabas, 1978	7.4	1.2	0.900	50
2.	Loma Prieta, 1989, Los Gatos	7.0	3.5	0.718	25
3.	Loma Prieta, 1989, Lex. Dam	7.0	6.3	0.686	40
4.	C. Mendocino, 1992, Petrolia	7.1	8.5	0.638	60
5.	Erzincan, 1992	6.7	2.0	0.432	21
6.	Landers, 1992	7.3	1.1	0.713	50
7.	Nothridge, 1994, Rinaldi	6.7	7.5	0.890	15
8.	Nothridge, 1994, Olive View	6.7	6.4	0.732	60
9.	Kobe, 1995	6.9	3.4	1.088	60
10.	Kobe, 1995, Takatori	6.9	4.3	0.786	40

The main response quantities of interest are acceleration of top storey and sliding displacement of isolator. Time-history plots of response of the example structure subject to Northridge (Rinaldi) 1994 excitation are shown in Fig. 3. The maximum response values are also shown. It is seen from Fig. 3(a) that there is substantial reduction in the maximum acceleration for structure isolated by VFPI, when compared to other isolation systems. These plots clearly demonstrate the effectiveness of VFPI in comparison with conventional FPS and PF system. From Fig. 3(b), it can be observed that the sliding displacements in case of VFPI may be more than PF system since the isolator force in VFPI can act either as restoring or driving force depending on the direction of motion, whereas in the PF system the constant frictional force always opposes the motion. However the residual displacements in VFPI are very small and are close to those of FPS, which clearly shows the effectiveness of the restoring mechanism. From these response characteristics, it is therefore found that VFPI retains the main advantages of both FPS and PF isolators.

The average, maximum and minimum response of the example structure subjected to ten near field ground motions has been presented in Table 3 for the three isolation systems. From these results it is seen that the maximum acceleration response of FPS is very large when compared to VFPI and PF systems for most of the excitations while the maximum sliding displacements in VFPI are large. However residual displacements for VFPI are very small and similar to that for PF system.

The energy quantities are better representation of structure response. Table 4 shows the input energy and conservative energy for the example system subjected to different ground motions. Input energy is a measure of effectiveness of isolation and conservative energy is the energy transmitted to the structure. The difference between the two is the energy dissipated through structural deformations. From the Table 4 it is observed that the input energy and conservative energy in case of FPS are both very large compared with FPS and PF systems. This shows that VFPI retains the effectiveness of PF system while significantly reducing the sliding and residual displacement when subjected to near-field ground motions.

Table 3: Response of example system for various near-field ground motion records.

	Maximum Structure Acc. (*g*)			**Maximum Sliding Displacement (*m*)**			**Residual Displacement (*m*)**		
	VFPI	FPS	PF	VFPI	FPS	PF	VFPI	FPS	PF
Ave.	0.184	2.515	0.168	1.197	0.650	0.757	0.440	0.067	0.465
Max.	0.222	13.33	0.200	2.404	0.989	1.037	2.251	0.284	0.834
Min.	0.144	0.364	0.134	0.470	0.305	0.528	0.000	0.000	0.078

Table 4: Energy characteristics of example system subjected to various near-field ground motion records.

	Input Energy ($\times 10^4$ N-m)			**Conservative Energy ($\times 10^4$ N-m)**		
	VFPI	FPS	PF	VFPI	FPS	PF
Ave.	30.47	270.08	23.36	4.74	177.87	1.39
Max.	46.60	721.80	38.84	14.97	516.00	5.08
Min.	17.75	114.80	11.66	1.19	28.25	0.47

6. CONCLUSIONS

The effectiveness of Variable Frequency Pendulum Isolator (VFPI) for vibration control of MDOF systems subjected to near-field ground motions has been investigated in this paper. A five-storey shear structure has been analysed for ten different near-field ground motions. From these investigations it is found that the VFPI is very effective in reducing the response of structures when compared to the FPS and PF isolators. The VFPI reduces the response substantially without losing the restoring capability thereby combining the advantages of both FPS and PF isolators.

Based on investigations of response of MDOF systems isolated by VFPI, the following conclusions can be drawn.

1. The VFPI is very effective in reducing the response of structures subjected to a large variety of near-field ground excitations. The performance of VFPI is found to be robust and superior to that of FPS and PF isolation systems.
2. VFPI includes effective energy dissipation as well as restoring mechanism.

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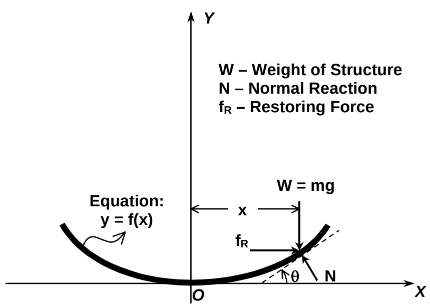

Figure 1: Free body diagram of sliding surface

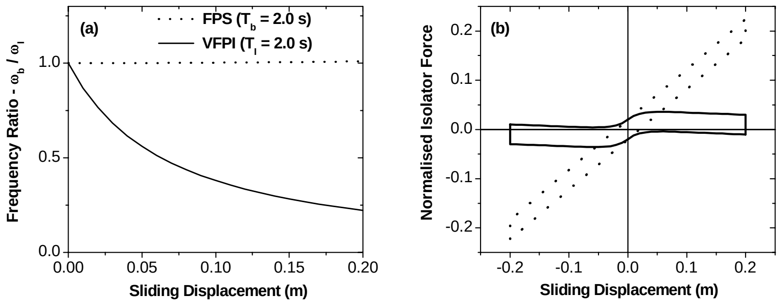

Figure 2: Frequency and restoring force characteristics of VFPI and FPS (a) Frequency ratio, (b) Normalized isolator force.

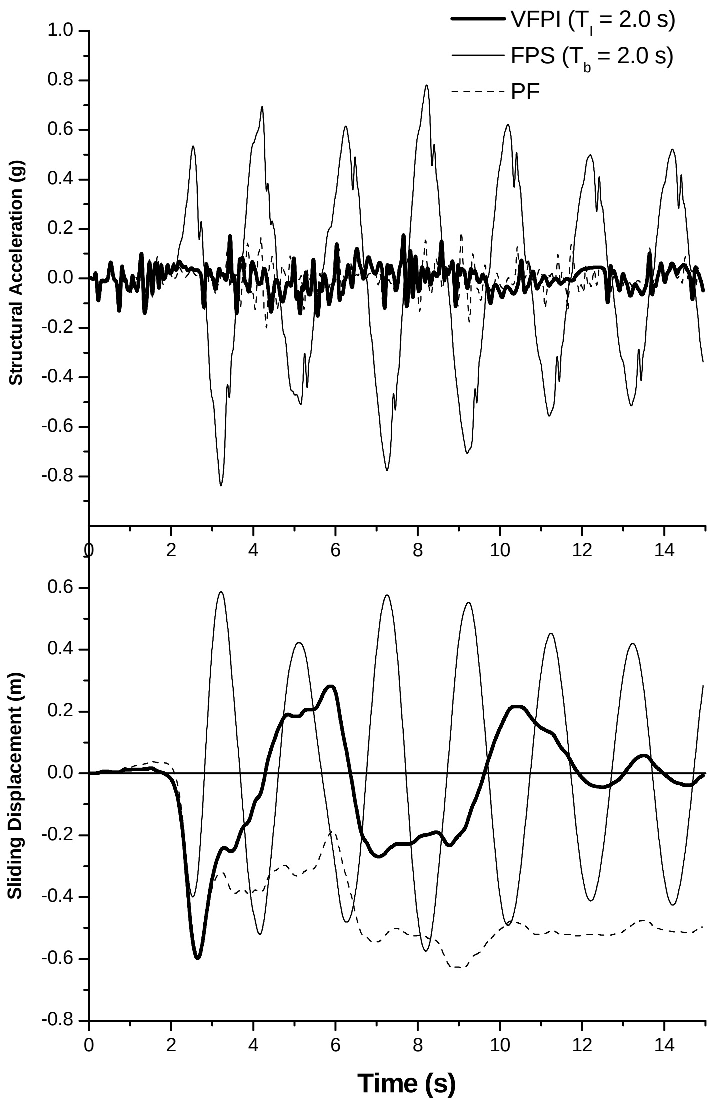

Figure 3: Partial time-history of response of structure subjected to 1994 Northridge (Rinaldi) ground motions.